THE CULTURAL IMPACT OF MATHEMATICS

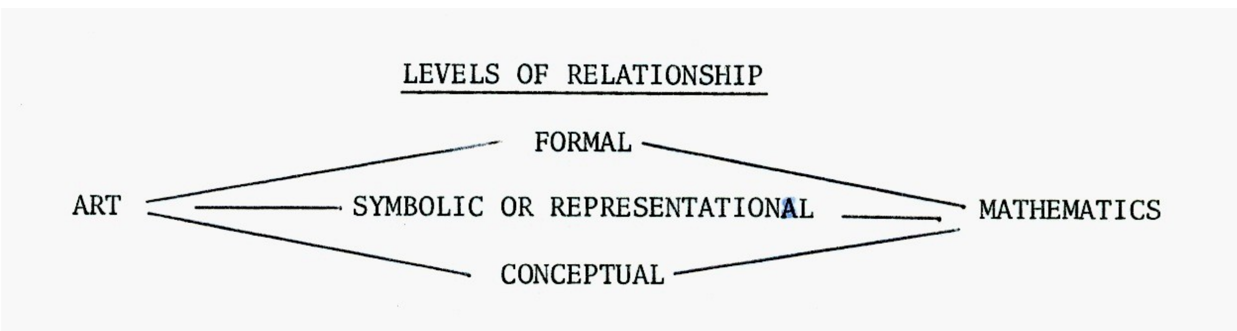
UNIT I : Mathematics and the Visual Arts

Chapter 4 - Summary and Abstraction

"The final abstract expression of every art is number."

Wassily Kandinsky, Concerning the Spiritual in Art

The studies of Raphael's The School of Athens and Duerer's Melancholia I proceeded with an eye to revealing the inherent relationships that may exist between art and mathematics in any work of art. These works were deliberately chosen because they are extraordinary in the rich detail they exhibit to express every level of this relationship.
It will be helpful to structure these levels for the purpose of discussion. They fall into three substrata:



At the formal level, we find that the mapping of an idea into its artistic space is ordered by a mathematically constructable framework. The mechanics of perspective, linear arrangement, symmetry, balance, proportion, color and color arrangement, etc., are employed at this level to carry out the ordering of content. Included in these formal elements are the mathematically derivable figures of design, as, for example, the S-curve of the Baroque period.

Taken together, they constitute what is commonly described as artistic style. As Janson[1] says: "In the visual arts, style means the particular way in which the forms that make up any given work of art are chosen and fitted together."

One of the major criteria for identifying the style of a work of art, whether it be historical or personal is the extent to which it can be analyzed into its structural components. As we have seen the language and content of mathematics plays an important role in this analysis. The lengthy survey of Chapter 1 can now be. primarily seen as an exercise in abstracting those geometric (formal) elements that have contributed to defining different styles.

Also in Chapter 1, there was frequent mention of the symbolic or representational functions of the geometric forms surveyed. The actual depiction of geometric figures is a frequent occurrence in all periods of art. They may appear, like the sphere and polyhedron in the Melancholia, as representative objects of geometry in their own right. Or, they may take on symbolic significance as commonly happens in religious art. For example, the hemispherical dome used in Byzantine architecture is directly linked to its symbolic portrayal of the "Dome of Heaven."
http://en.wikipedia.org/wiki/Byzantine_architecture

Beyond the formal and symbolic or representational levels, mathematical ideas enter into a work of art on a conceptual level. As we shall see later, this is the most general level of abstraction that can be attained in the relationship between art and mathematics. At this level, the intrinsic meaning of the work of art is given mathematical substance by this relationship. An excellent example is the analysis we made of the contents of the slate held before Pythagoras in The School of Athens. The cosmic harmony reflected in the entire fresco is a consequence of the Renaissance acceptance of the universal application of Pythagorean music theory. Thus, a conjunction is established between the philosophies of art and mathematics at the conceptual level.

Sometimes all three levels can be built into a work simultaneously. For example, the spheres held by Zoroaster and Ptolemy in The School of Athens are models of the Earth and sky. Together they symbolize Ptolemy's geocentric theory of the universe, a mathematical structure that stood as the foundation of astronomy throughout the Renaissance period. At the formal level, we found the circle (the analogous form of the sphere in two dimensions) to be a key in organizing the composition of the fresco. Contrast this with the "Dome of Heaven" mentioned above. It's conceptual significance is theological, not mathematical.

In Duerer's Melancholia, the conceptual relationship is implied first by the artist's personal identity with the practice of mathematics (i.e. his theoretical treatises and his invention of the magic square in the engraving). Then, on a deeper conceptual level, there is the spiritual conflict between the intuitive-mystical and deductive-rational manifestations of the human mind. The engraving is laden with mathematical objects having symbolic meaning. Finally, at the formal level, the rabatment of the shorter side on to the longer formed a framework that determined the placement of the figures and the organization of perspective. Again, this framework was shown to be related to the magic square.

The conceptual level is the most difficult to ascertain. Consider, for example, Botticelli's portrait of St. Augustine.
http://en.wikipedia.org/wiki/File:Sandro_Botticelli_050.jpg

Without the presence of astronomical instruments and a mathematical textbook, we would not have a clue to the contemplations of this great Doctor of the Church. One thousand years of history separates the artist from his subject. However, with a little research we can learn that St. Augustine attempted to reconcile Platonism with the Bible. He was fascinated with Pythagorean mathematics, particularly its mystical aspects, befitting a theologian. His comments on the perfect number six are worth quoting in this respect. The reason for calling six a "perfect number," by the way, lies in the fact that it is equal to the sum of its proper factors, I + 2 + 3. Here is what St. Augustine had to say about it: "Six is a number perfect in itself, and not because God created all things in six days; rather the inverse is true, that God created all things in six days because this number is perfect, and it would remain perfect, even if the work of the six days did not exist."[1]

Again, as an echo of our previous discussion of number mysticism, Sullivan points out, "From speculations of this sort the Pythagorean doctrine developed, on the one hand, in a thoroughly respectable philosophic manner into the doctrine of necessary truths, and on the other descended to cabbalistic imbecilities."[2]

Given our knowledge of Renaissance art and philosophy, we would expect Botticelli's portrait of the scholarly Saint to focus on his Platonic interests, since the artist himself was deeply involved in the revival of Platonism. One might argue with good cause that the world of painting and sculpture contains very few examples like this, where the subject matter is explicitly connected to mathematical concepts. In the main we find that most works of art do not relate to mathematics beyond the formal or symbolic levels. However, there are some notable exceptions. The miracle of the

Gothic cathedral is the direct result of Pythagorean concepts about the harmony of the universe. These ideas were filtered through the theology of the man who started the Gothic style, Abbot Suger of the Abbey of St. Denis. When we enter the cathedrals of Chartres or Notre Dame, we are enclosed in a world which symbolically represents the source of “Divine Reason”. http://en.wikipedia.org/wiki/Notre_Dame_Cathedral

Geometry is the realization of “Divine Reason” and it strictly determined every detail of these fantastic structures. The raising of the walls by shifting their pressures to exterior flying buttresses not only symbolized man's aspiration to God; it also permitted the infiltration of “Divine Light” to illuminate the vast interior space. In a monumental way these scientifically conceived structures echo the words of the Scholastic philosopher, Peter Abelard: “I must understand in order to believe."

It is possible even in religious art, therefore, to encounter a conceptual relationship between the artistic product and mathematics. This is underscored by the figure of God as Geometer. http://commons.wikimedia.org/wiki/File:Blake_ancient_of_days.jpg

With the coming of the scientific revolution more and more artists have been stimulated by the subject matter of the sciences. The fascination with scientific and technological objects has resulted in a number of outstanding works reflecting this advance in human knowledge. http://en.wikipedia.org/wiki/A_Philosopher_Lecturing_on_the_Orrery

In our own time, the graphic art of Maurits Escher has had a special interest for mathematicians and his works have enjoyed a considerable popularity in the United States. His woodcuts and lithographs combine a child-like fantasy world with an amazing intuitive grasp of advanced mathematical concepts. http://en.wikipedia.org/wiki/M._C._Escher

Escher is principally known for his multi-perspective effects, the covering of plane surfaces by interlocking figures, and figures which transform themselves from two to three dimensions. The subject matter of his works reads like a catalog of modern mathematics; such things like topology, projective geometry, hyperbolic geometry, isometries, and differential geometry. Most of them are joined with organic forms to be found in the natural world, abstracted to fit into his mathematical fantasies. Escher is however, unique in style. His work is so specialized and personal that it does not lend itself to the development of a stylistic trend. It stands as a fascinating, though isolated, example of the conceptual relationship between art and mathematics.

More important for the development of art in general is the connection between mathematics and the movements which have become known as "abstract" and/or “nonobjective" art. In the written literature about art, no idea seems to draw more upon the language of mathematics than that of abstraction. The historian of art, Janson writes: " ... the term is usually taken to mean the process (or the result) of analyzing and simplifying observed reality. Literally, it means 'to draw away from, to separate'.”[3]

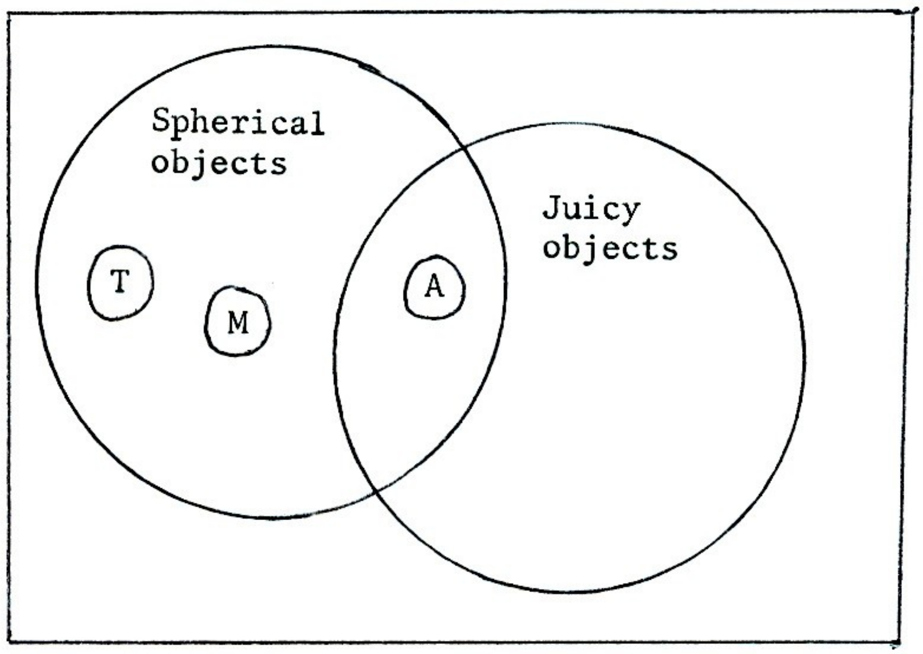
To clarify this definition, he uses the example of number: "If we have ten apples and then separate the ten from the apples, we get an 'abstract number,' a number that no longer refers to particular things.” He then goes on to point out that the word 'apples' is also an abstraction since we are talking about a class (or set) of things rather than an individual member of that class.

http://en.wikipedia.org/wiki/File:C%C3%A9zanne,_Paul_-_Still_Life_with_a_Curtain.jpg

The underlying conception in both instances is that of set membership. Numbers are defined in terms of sets. There is a standardized set called ten. If we have a set of ten objects like apples, we can put this set into a one-to-one correspondence with the standard set so that each apple is matched up with a single element of the standard set. The standard set is called a subset of the set of natural or counting numbers. Thus, the standard set, ten, consists of the first ten members of the set of natural numbers.**4** The natural numbers always answers the question of "how many" for us. The question of cardinality (i.e. how many) is a much simpler one to answer in terms of sets than that of 'apple-ness.' What are the characteristics that make an apple distinctive?

If we begin by saying an apple is spherical, then it shares a property that makes it a member of the same set as tennis balls, the Moon, and peaches. If apples are defined next as juicy, the intersection of the set of juicy objects with the set of spherical objects will eliminate the tennis balls and the Moon. It will also make the concept of apple-ness more specific and less abstract. In other words as the apple is assigned membership in more and more well-defined sets we get closer and closer to describing its apple-ness but further away from its ability to represent general concepts.

Venn diagrams.



Key

A = set of apples

T = set of tennis balls

M = set of moons

Eventually this process of intersection will isolate the set of apples from all other objects. Each object in this set will be a representative of the entire set. It may be wormy, sour, green, or flecked with bruises, but it will still be uniquely apple-like. As soon as one object can represent all others in the set we have reached the bottom level of abstraction. Of course, it is possible to separate out the sub-set of green apples and abstract this set by a representative in the same way; the process is limitless in this direction. But the process is the same, it essentially creates the distinction between the individual object and the set to which it belongs.

Artistic abstraction moves in the opposite direction. It strives for greater generalization and larger meaning. It asks how much significance can an apple bear without becoming meaningless. Consider Cezanne's still-life in the link above. As Janson points out: " ... this quest for the 'solid and durable' can be seen even more clearly, Not since Chardin have simple everyday objects assumed such importance in a painter's eye . ... the forms are deliberately simplified, and they are outlined with dark contours; ... When Cezanne takes these liberties with reality, his purpose is to uncover the permanent qualities beneath the accidents of appearance (all forms in nature, he believed, are based on the cone, the sphere, and the cylinder). This order underlying the external world was the true subject of his pictures, but he had to interpret it to fit the separate, closed world of the framed canvas."**5**

Cezanne is considered by many authorities to be the father of modern abstraction. However, we can trace this conscious and deliberate process back to the Renaissance artist, Piero della Francesca about whom Janson writes : "This mathematical outlook permeates all his work. When he drew a head, an arm, or a piece of drapery, he saw them as variations or compounds of spheres, cylinders, cones, cubes, and pyramids, endowing the visible world with some of the impersonal clarity and permanence of stereometric bodies. We may call him the earliest ancestor of the abstract artists of our own time, for they, too, work with systematic simplifications of natural forms."[6]

http://en.wikipedia.org/wiki/File:Piero_-_The_Flagellation.jpg

Paradoxically this tendency to abstraction in Piero's work grew out of his dedication to scientific perspective, the most systematic method for accurately representing visual reality that has ever been devised. Perspective is a generalized technique for creating the illusion of three dimensional space on a two dimensional surface. Piero's stereoscopic style was a direct outcome of his efforts to model his figures in a "natural three dimensional manner."

It is not a contradiction to say that perspective is one of the greatest achievements of abstraction in the art of painting. No more efficient system of "analyzing and simplifying observed reality" has ever been formulated. Its existence as a mathematical generalization of how we see has been confirmed by photography

http://en.wikipedia.org/wiki/File:Pont_du_gard.jpg

and rigorously defined in the structure of projective geometry. What began as an artistic theory has culminated in one of the glorious pages in the history of mathematics.

As far as we know, the paintings by Filippo Brunelleschi (1377- 1446) of the Florentine Piazza were the first unequivocal applications of the rules of what is called scientific perspective.

http://en.wikipedia.org/wiki/Brunelleschi

The system is also referred to as artificial or focused perspective. This technique of creating the illusion of depth on a two dimensional surface was 200 years in the making. Over the millenia many attempts had been made to create the illusion of depth on a plane surface, but as far as we know there was no generalized system established before the Renaissance. It finally found its theoretical foundations in a treatise Della Pittura (1435) by Alberti (1404-1472), the architect who was noted earlier for his advocacy of the musical consonances in the visual arts.

http://en.wikipedia.org/wiki/Leone_Battista_Alberti

http://en.wikipedia.org/wiki/De_Re_Aedificatoria

To summarize the features, White writes: "The four characteristics (of artificial perspective) are that: (a) There is no distortion of straight lines. (b) There is no distortion, or foreshortening of objects or distances parallel to the picture plane, which is therefore given a particular emphasis. (c) Orthogonals converge to a single vanishing point dependent on the fixed position of the observer's eye. (d) The size of objects diminishes in an exact proportion to their distance from this observer, so that all quantities are measurable. The result is an approximation to an infinite, mathematically homogenous space, and the creation of a new, and powerful means of giving unity to the pictorial design."**7**

http://en.wikipedia.org/wiki/Linear_perspective

To nail down the technique, it was common for Renaissance painters to use lined pavements converging to the vanishing point.

http://en.wikipedia.org/wiki/File:Perugino_Keys.jpg

In later periods, artists were able to integrate the construction lines controlling the perspective in their preliminary drawings, so that we are not so conscious of the technique being used in the final painting.
http://en.wikipedia.org/wiki/File:Paolo_uccello,_studio_di_vaso_in_prospettiva_02.jpg

To think of perspective as an abstraction is a paradox today because we are conditioned to view abstraction as a drawing away from reality. Perspective may be said to have lost its own "reality" as an abstract mathematical system, having been drawn into the total aesthetic of realism which it engendered. The use of perspective in evoking the natural world was admirably captured in Dutch paintings of the 17th century. When we view a painting employing focused perspective, the illusion of being able to step into the world it contains is successfully achieved. It is this sense of depth that coincides so closely with our feelings of the way things are in our visual perception of the world around us. When this is combined with a geometric sensitivity to the diffusion of light, as it is in the works of Vermeer, the effect of reality is uncanny.
http://en.wikipedia.org/wiki/File:Jan_Vermeer_van_Delft_011.jpg

Yet, we must also be conscious of the fact that both the painting and the way we see things distributed in space are unreal; simply the mind's registration of the mechanical functioning of our visual apparatus. After all, objects don't actually grow smaller as they recede in distance from the observer. From this point of view, the movement we call "realism" in art would be more accurately described as "perceived realism" and, as science has proved so frequently with respect to our sense perception, it may have no foundation in reality at all. Indeed, conscious abstraction in the arts of today may be viewed as a search for realities beyond perception, the same kind of pursuit that is present in contemporary scientific research. It is for this reason that artistic abstraction has become the antithesis of the perceived realism produced by perspective.

Consider now a painting by another Dutch artist, Piet Mondrian (1872-1944). At first glance, one might question whether this was a decorative design or an illustration for a lesson in geometry. This painting is one of a long series of compositions he did restricting himself to the same basic elements of lines, rectangles, and colors.

http://en.wikipedia.org/wiki/File:Mondrian_Composition_II_in_Red,_Blue,_and_Yellow.jpg

If one is left alone with this work for a period of time, the serious viewer would begin to ponder the basis of its form and composition. Why did Mondrian place that red there and how did he determine just this arrangement of rectangles and no other? What comes into play in the mind are questions of balance, proportion, symmetry, and contrast. In other words we are forced to consider the universals that characterize all art without cluttering our minds with the specifics of representational meanings.

What has Mondrian accomplished in the tightly controlled and limited world of his canvas? He confronts us, like a modern day Descartes, rejecting all of the traditional knowledge of his time to ask, "What is truth?" Descartes (I596-1650), who is considered to be the first modern philosopher of science, found that he could accept the axioms and theorems of Euclidean geometry as truths because, he believed, mathematical knowledge existed independently of human perceptual knowledge. From this foundation, he arrived at his famous axiom, "cogito ergo sum," I think therefore I am, his basic confirmation of self-existence in the face of universal doubt.

It is the same with Mondrian. He tells us in words, elsewhere, what his paintings express.

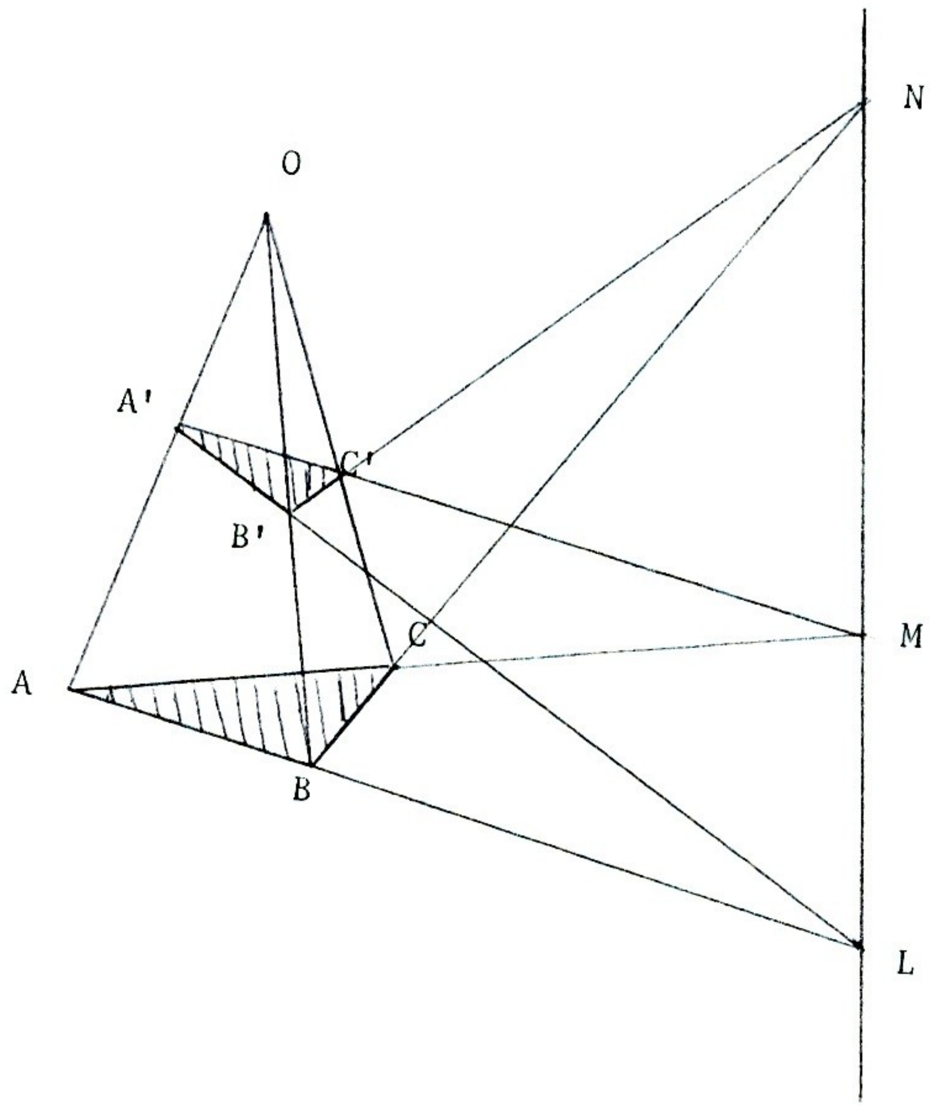
http://en.wikipedia.org/wiki/Piet_Mondrian

He was seeking truth in a form he described as "pure reality." He defined this as equilibrium "through the balance of unequal but equivalent opposition.” His objective was to share in that same penetrating glimpse of pure beauty that was immortalized in the magnificent sonnet by Edna St. Vincent Millay:

Euclid alone has looked on Beauty bare.
Let all who prate of Beauty hold their peace,
And lay them prone upon the earth and cease
To ponder on themselves the while they stare
At nothing, intricately drawn nowhere
In shapes of shifting lineage; let geese
Gabble and hiss, but heroes seek release
From dusty bondage into luminous air.
O blinding hour, O holy, terrible day,
When first the shaft into his vision shone
Of light anatomized! Euclid alone
Has looked on Beauty bare. Fortunate they
Who, though once only and then but far away,
Have heard her massive sandal set on stone.

To reach this supreme level of abstraction, Mondrian stripped away every element that could relate his compositions to representational ideas. Michel Seuphor, his biographer, writes: "The significance of Mondrian's art consists primarily in his search for purity of form through reducing its means of expression to a simple statement of relationship, the essential relationship being created by two straight lines meeting at right angles. Not only curved lines and spatial illusion, but also any indication of brushwork and any treatment reminiscent of Impressionist technique were rejected. The result was a sort of mystical pursuit of the absolute, which swept away the world of appearances. Since the picture surface is a plane, the painting on it should also be a strict plane and as a result becomes the means of communicating a conception, bound neither to time nor space, but concerned only with relationship. However, Mondrian always tried to express this relationship dynamically, in other words, by an asymmetrical equilibrium."[8]

Euclid, then, has not been alone in looking on "Beauty bare." Artists like Mondrian have caught a glimpse of her passing. However, from a mathematical viewpoint, She is still clad in veils of abstraction which have but slowly been removed. Two thousand years after Euclid, the 17th century architect and engineer, Girard Desargues, published a small book with the purpose of simplifying the rules of perspective for practicing artists. In the process he laid the foundations for projective geometry, a branch of mathematics, so general in approach, that it would eventually encompass the whole of Euclidean geometry and the non-Euclidean geometries as well that weren not discovered until the 19th century. The next figure illustrates his famous theorem which relates projection and section.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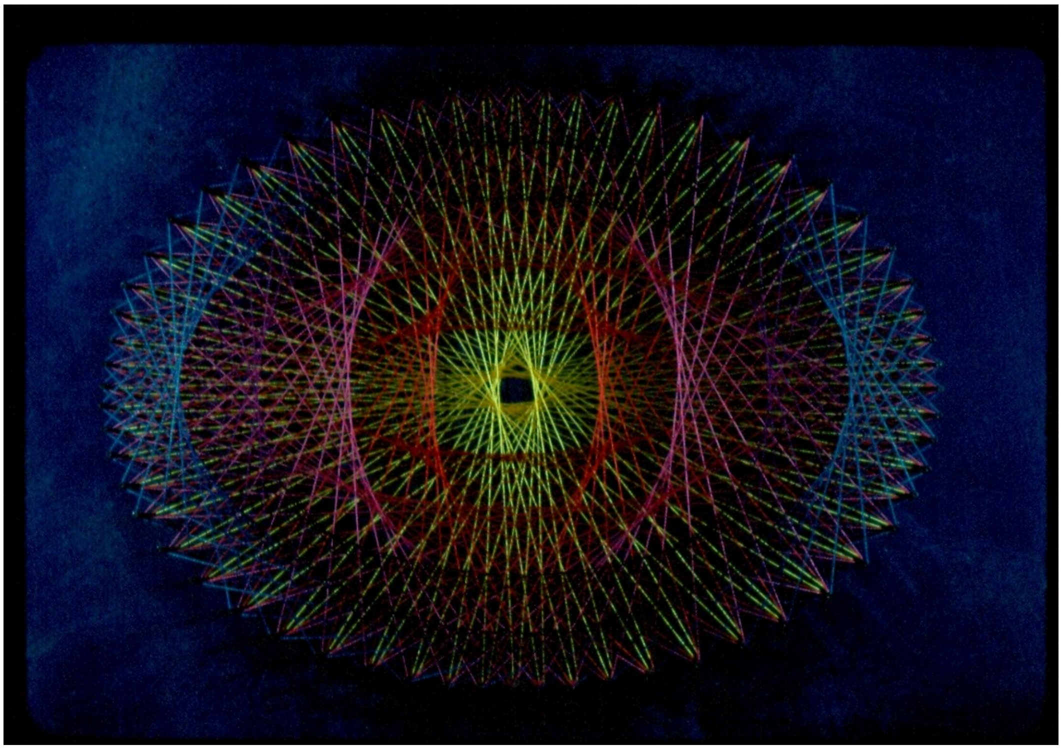
Assume an observer is located at point 0 directly over a plane containing triangle ABC. If a plane of glass is passed between the observer and the plane containing triangle ABC, and it is tilted at an angle, the section of the projection of triangle ABC on the glass plane will be seen as triangle A'B'C'. If the corresponding line segments of the two triangles are extended, Desargues asserted they will meet in three points located on the same line. This assertion remains true even if the triangles lie in the same plane. Thus, out of what began as a 'recipe' to allow the artist complete flexibility in establishing the viewer's angle of observation consistent with the rules of perspective, comes a generalization that is extendable to all space.

Like Mondrian's compositions, the "pure beauty" of Desargues' theorem does not reside in the confluence of lines and points that are perceived in the diagram. On the contrary, it lies in the unadorned relationship that exists independently as an idea and that transcends its visual realization.

Once we have entered the realm of thought, the possibilities become limitless. For example, projective geometry shares with other branches of mathematics an even higher level of abstraction, one of the most beautiful in all of philosophy as well, the Principle of Duality.

To illustrate this, look at a curve tracing made from colored thread. In analytic geometry (the invention of Descartes that joined algebra and Euclidean geometry), an ellipse is defined as a set of points that satisfy a specific algebraic relationship. Projective geometry allows for the same curve to be defined by a set of lines, all of which have the property of being tangent to the curve. It is this definition which is reflected in the construction.



The Principle of Duality, the Yang and Ying of mathematics, operates in projective geometry so that the mere exchange of the word "line" for "point" in a theorem produces an equally valid conclusion backed up by an equally logical proof. Even Desargues' theorem is accessible to this exchange, producing a valid dual counterpart. When this was discovered in the 19th century, Beauty's "massive sandal set on stone" resounded throughout the mathematical world.

We have made the point by analogy that the pure abstractions generated by Mondrian's compositions and Desargues' theorem share a common essence of Beauty. It would be erroneous, however, to apply the Principle of Duality to these products of artistic intent and mathematical deduction. They are not interchangeable even as ideal forms in the Platonic sense. Mondrian's works are not symbolically or conceptually related to mathematics, except by analogy. Formally, we can perceive the obvious geometric figures that make up his works and the not so obvious relationships, such as the golden section, which organize some of the figures.

However, the intrinsic meaning of his compositions are as personal and unique as those of his fellow countryman, Rembrandt. It comes down to this; that no matter how abstract a work of art may be, it cannot break the bounds of identification with its creator. Mondrian's style has been imitated by countless artists, but no one has ever been successful in passing off a forgery of one of his works. Whatever generalizations he arrived at in his search for "pure reality",they are artistic ones.

They have been assimilated into the main stream of artistic development in this century and have found their most important application, not in painting, but in architecture. http://en.wikipedia.org/wiki/Seagram_Building

Like Descartes, Mondrian may have used mathematics as a model but the end product was art; a deductive art it may be allowed, but as intuitive and personal as Descartes' philosophy.

Having looked at the evidence, it must be concluded that abstraction in art is primarily related to mathematics at the formal level. Abstraction, then, is a product of style and what makes a style distinctive is the consistent selection of particular visual elements to create a unified whole. Whether it be the rigid 90^{o} orientation of head to torso in ancient Egyptian reliefs,
http://en.wikipedia.org/wiki/File:Sunken-relief1.jpg
or the conscious dissolution of line in the color compositions of the Impressionists,
http://en.wikipedia.org/wiki/File:Edgar_Germain_Hilaire_Degas_072.jpg
these are the elements of composition which put the stamp of uniqueness on works of art, be they representative of an historical period or the output of a single artist.

We have seen that perspective is an abstraction, one that had an all-pervading influence for over 400 years. From the Renaissance artists to some of the leading artists of our own time,
http://en.wikipedia.org/wiki/File:Christinasworld.jpg
it has persisted as a commonly accepted element of style for so long that its deliberate denial became a formal requirement for those who would achieve a new system of organization for two dimensional space. The revolution began with Cezanne in the closing years of the nineteenth century. It arrives in the twentieth century like the burst of a bombshell in the works of Pablo Picasso (1881-1973).
http://en.wikipedia.org/wiki/Picasso

It is almost sacrilegious to speak of the relationship between mathematics and the visual arts in his presence. He engaged his art with such complete freedom and spontaneity that he is considered by many to be the most intuitive and original artistic genius of his century. He was perhaps the most prodigious inventor of stylistic elements that has ever lived.

Unlike Mondrian, Picasso's abstractions were stimulated by objects in the natural world. His early works display complete mastery of the expressive modeling of the human form , a product of the traditional Spanish heritage from Goya onwards.

http://en.wikipedia.org/wiki/File:Femme_aux_Bras_Crois%C3%A9s,_Picasso.jpg

In 1900 he came to Paris and fell under the influence of both Cezanne and the primitive African sculptures that were then being exhibited.

http://en.wikipedia.org/wiki/File:Ife_sculpture_Inv.A96-1-4.jpg

The result was his invention with Braque (1882-1963) of cubism. A typical example of this style shows the most prominent features are the emphasis on basic geometric forms, the intersection and overlapping of planes, and the simultaneous views of objects from different angles, and the fragmentation of forms into prismatic facets.

http://en.wikipedia.org/wiki/File:Picasso_three_musicians_moma_2006.jpg

Originally monochromatic to emphasize its architectural elements, colors from the spectrum were introduced to blend in with the harmony of the basic architecture. At a later stage in his development of these ideas, individual objects emerge, retaining the distortions imposed upon them by the stereoscopic conception of space.

http://en.wikipedia.org/wiki/File:Picasso_Portrait_of_Daniel-Henry_Kahnweiler_1910.jpg

The great mural of 1937, Guernica , shows how versatile this style can be in expressing the horrors wrought on a civilian population by modern warfare. The figures are massed, twisted, and distorted into one agonizing primal scream. A new geometry is being exploited here, topology, which raises the question of what remains invariant under distortion.
http://en.wikipedia.org/wiki/File:PicassoGuernica.jpg

It's interesting to reflect, in connection with Guernica, that the most recent applications of topology is in a field of study called Catastrophe Theory. A vivid example of this theory is shown in the study of wake vortex effects.
http://en.wikipedia.org/wiki/Chaos_theory
http://en.wikipedia.org/wiki/File:Airplane_vortex_edit.jpg

Picasso's works must be seen as a whole to appreciate its complete unpredictability. They refuse to lend themselves to analysis. Yet, in spite of what appears to be chance elements, intended to shock or delight, one always feels they have been placed in his compositions with the unerring hand of a master. His intuitive sense of balance and proportion was as keen as Mondrian's. Picasso was blessed with the rarest of human qualities, an innate ability to show us a world that has never before existed.

It seems to be a fact of life that artists with a predilection for abstraction tend to cast an envious eye on their creative counterparts in music.

Stripped of literary or pictorial associations, music has long been considered the most abstract of the arts and, at the same time, the most immediate and direct in its emotional impact. The reasons for this will be explored in Unit II. However, it is revealing for the present discussion to examine the influence that music has had on the visual arts, particularly in the 20th century.

In a letter to a fellow artist, Franz Marc (1880-1916) wrote the following after an evening of Arnold Schoenberg's chamber music:
"Can you imagine music in which tonality has been completely abandoned! I was reminded constantly of Kandinsky's large compositions."**9**
http://en.wikipedia.org/wiki/Kandinsky
The following link will indicate something of the associations Marc had in mind. http://en.wikipedia.org/wiki/File:Kandinsky-Blue_Rider.jpg
Prior to World War I , the ideas about abstraction in the visual and musical arts were germinating. Schoenberg was closely related to the publications of the Blue Rider group formed by Kandinsky and his fellow artists like Marc. In the period between the two World Wars, these ideas came to fruition.
http://en.wikipedia.org/wiki/File:Kandinsky_WWI.jpg
http://en.wikipedia.org/wiki/File:Kandinsky_white.jpg

As Roh writes: "Kandinsky's nonobjective painting may be best understood as a form of dynamic expressionism, a movement of inner feeling that fought to escape the limiting effect of the fixed object."**10**

The key phrases in this quote are 'dynamic expressionism' and 'movement of inner feeling.' Music is the basic art for capturing the essence of form in motion. It is the ultimate dynamic expression. Kandinsky invented forms and juxtapositions that visually generated a sense of movement analogous to music. In his early abstractions like that of, we find brushed curves that swirl and spiral, absorbing the viewer into a microscopic world of indistinct organic forms that are swept into a bubbling "primordial soup." In a later period, Kandinsky returns to the simplified forms of plane geometry. "Instead of an irrationally flowing diagram we have strict regulation. Clear colors confront each other additively, apparently aiming at a constructivist statement wherein highly individualized visual tensions are achieved."**11** In music, tension is at the heart of contrast between consonance and dissonance. Harmonic rhythms are propelled by this interplay. Although there is a classical austerity to this second phase of Kandinsky's growth in abstraction, the sense of movement is maintained.

At the end of his life, Kandinsky softened this objectivity with a more lyrical quality. He employed " ... large, irrationally playful forms, bold color contrasts, and radiant symbols .. ,"**12** creating long graceful melodic lines.

Although Kandinsky succeeded in suggesting musical qualities in his works, they still remain faithful to the art of painting. "In his theories he constantly introduces analogies to absolute music, but he did not rob painting of its unique nature, for he never rejected the specific properties that do not belong to music--line, color, and space..”13

The ultimate realization of Kandinsky's "dynamic expressionism" reached its climax in the so-called "action paintings" of Jackson Pollock (1912-1956). http://en.wikipedia.org/wiki/Jackson_Pollock

In Pollock's works, the artist becomes performer as well as composer. He literally threw himself into his compositions by placing his large canvases on the floor of the studio, traversing them with a funnel dripping different colored paints, charging his surface with rapid gestures that involved his entire physical, emotional, and intellectual being in the process. The result is that each picture area becomes a composition in linear rhythms that are " ... organic, never decorative.”**14**

http://en.wikipedia.org/wiki/File:No._5,_1948.jpg

One might call him the first "biorhythmic" painter. It is easy to look at

Pollock's improvisational techniques as a kind of hoax, "giving free play to chance and instinct or an apology to amateurism."[15] This criticism is unjustified, for "it is easy to show the strict cohesion of his work, which was the result of an exceptional craftsmanship, virtuosity and self-discipline."[16]

To be successful, improvisation in any art form must emanate from a reservoir of mastery in the craft of composition. The improviser must be guided by an inner sense of the rightness of fit for the stylistic elements that are selected. There is no question but that Pollock had this gift, for in his works the act of creation and the results became one.

One cannot help but wonder in viewing Pollock's works, just how far mathematical ideas are capable of penetrating such a highly subjective ordering. We are confronted here with the internal mathematics of human physiology and psychology. It's interesting to note in this respect, that nonobjective art has been successfully used for making contact with patients in psychotherapy. Is it because the content is so elemental that it can communicate with the subconscious mind?

Science and mathematics are not yet capable of answering this question. We have reached the outer boundaries between art and mathematics where the intersection between analysis and synthesis may be the empty set. Pollock represents the heroic tradition. He put all the resources of his genius to visually capture the rhythms of life itself; the continuities and discontinuities, the accelerations and retards, the ever-changing pulsations of the human condition.

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